The mission of the Chardon Local Schools is high achievement for all students where learning is our most important work.

Course of Study — MATH

Revised November 2021

MULTIVARIABLE CALCULUS

CHARDON LOCAL SCHOOLS

#WeAreChardon

COS — MATH — ***Revised November 2021***

Multivariable Calculus

Strand: Vectors and Geometry of Space

Learning Standard:	**How Taught?**
• Write the component form of a vector • Perform vector operations and interpret the results geometrically • Write a vector as a linear combination of standard unit vectors. • Use vectors to solve problems involving force or velocity • Understand the three-dimensional rectangular coordinate system. • Analyze vectors in space. • Use three-dimensional vectors to solve real-life problems. • Use properties of the dot product of two vectors. • Find the angle between two vectors using the dot product. • Find the directional cosines of a vector in space. • Find the projection of a vector onto another vector. • Use vectors to find the work done by a constant force. • Find the cross product of two vectors in space. • Use the triple scalar product of three vectors in space. • Write a set of parametric equations for a line in space. • Write a linear equation to represent a plane in space. • Sketch the plane given by a linear equation. • Find the distance between points, planes and lines in space • Recognize and write equations for cylindrical surfaces. • Recognize and write equations for quadratic surfaces. • Recognize and write equations for surfaces of revolution. • Use cylindrical coordinates to	**Teaching activities may include, but are not limited to:** • Direct Instruction • Cooperative Groups • Stations • Data Driven Instruction • Scaffolding

Multivariable Calculus

represent surfaces in space.
- Use spherical coordinates to represent surfaces in space

Materials:
- Texas Instrument Graphing Calculator
- Geogebra 3D Calculator
- Chromebook
- Quizlet
- Kahoot
- Quizizz

How Assessed?
Assessments may include, but are not limited to:
- Pre-Assessments (pre-tests, observation, anticipation guide, questioning, diagnostics)
- Formative Assessments (entry/exit slips, group work, reflections, discussions, writer's workshops, homework/classwork, self and peer evaluations, observations, conferences, rubrics)
- Summative Assessments (formal essays, using rubrics; tests/exams, projects, creative assignments, presentations)

How Re-Taught?
Re-teaching activities may include, but are not limited to:
- breaking down concept into smaller components
- presenting the information again in a different way
- Universal Design for Learning principles offering students opportunities to experience and engage material in new and different way
- practice activities such as computer tutorials, games, hands-on activities
- review sessions

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Multivariable Calculus

Strand: Vector Valued Functions

Learning Standard:

- Analyze and sketch a space curve given by a vector-valued function.
- Extend the concepts of limits and continuity to vector-valued functions.
- Differentiate a vector-valued function.
- Integrate a vector valued function.
- Describe the velocity and acceleration associated with a vector-valued function.
- Use a vector-valued function to analyze projectile motion.
- Find the unit tangent vector at a point on a space curve.
- Find the tangential and normal components of acceleration.
- Find the arc length of a space curve.
- Use the arc length parameter to describe a plane curve or space curve.
- Find the curvature of a curve ata point on the curve.
- Use a vector-valued function to find frictional force.

How Taught?

Teaching activities may include, but are not limited to:

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- Cooperative Groups
- Stations
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Multivariable Calculus

	● Summative Assessments (formal essays, using rubrics; tests/exams, projects, creative assignments, presentations)
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Strand: Functions of Several Variables

Learning Standard:	How Taught?
• Understand the notation for a function of several variables. • Sketch the graph of a function of two variables. • Sketch level curves for a function of two variables. • Sketch level surfaces for a function of three variables. • Use computer graphics to graph a function of two variables. • Understand the definition of a neighborhood in the plane. • Understand and use the definition of the limit of a function of two variables. • Extend the concept of continuity to a function of two variables. • Extend the concept of continuity to a function of three variables. • Find and use partial derivatives of a function of two variables • FInd and use partial derivatives of a function of three or more variables. Find higher order partial derivatives of a function of two or three variables. • Understand the concept of differentiability to a function of two variables. • Use a differential as an approximation. • Use the Chain Rule for functions of several variables. • Find partial derivatives implicitly. • Find and use directional derivatives of a function of two variables. • Find the gradient of a function of two variables. • Use the gradient of a function of two variables in applications. • Find directional derivatives and gradients of functions of three variables. • Find equations of tangent planes and	**Teaching activities may include, but are not limited to:** • Direct Instruction • Cooperative Groups • Stations • Data Driven Instruction • Scaffolding

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normal lines to surfaces. • Find the angle of inclination of a plane in space. • Compare gradients $\bar{\nabla} f(x, y)$ and $\bar{\nabla} F(x, y, z)$ • Find absolute and relative extrema of a function of a function of two variables. • Use the Second Partials Test to find relative extrema of a function of two variables. • Solve optimization problems involving functions of several variables. • Use the method of least squares. • Understand the method of Lagrange Multipliers. • Use Lagrange multipliers to solve constrained optimization problems. • Use the Method of Lagrange Multipliers with two constraints.	
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Strand: Multiple Integration

Learning Standard:

- Evaluate an iterated integral
- Use an iterated integral to find the area of a plane region.
- Use a double integral to represent the volume of a solid region.
- Use properties of double integrals.
- Evaluate a double integral as an iterated integral.
- Write and evaluate double integrals in polar coordinates.
- Find the mass of a planar lamina using a double integral.
- Find the center of mass of a planar lamina using double integrals.
- Find moments of inertia using double integrals.
- Use double integrals to find the area of a surface.
- Use a triple integral to find the volume of a solid region.
- Find the center of mass and moments of inertia of a solid region.
- Write and evaluate a triple integral in cylindrical coordinates.
- Write and evaluate triple integrals in spherical coordinates.
- Understand the concept of a Jacobian.
- Use a Jacobian to change variables in a double integral.

How Taught?

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- Pre-Assessments (pre-tests, observation, anticipation guide, questioning, diagnostics)

Multivariable Calculus

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Strand: Vector Analysis

Learning Standard:	How Taught?
• Understand the concept of a vector field. • Determine whether a vector field is conservative. • Find the curl of a vector field. • Find the divergence of a vector field. • Understand and use the concept of a piecewise smooth curve. • Write and evaluate a line integral. • Write and evaluate a line integral of a vector field. • Write and evaluate a line integral in differential form. • Understand and use the fundamental Theorem of Line Integrals. • Understand the concept of independence of path. • Understand the concept of conservation of energy. • Use Green's Theorem to evaluate a line integral. • Use alternate forms of Green's Theorem. • Understand the definition of and sketch a parametric surface. • Find a set of parametric equations to represent a surface. • Find a normal vector and a tangent plane to a parametric surface. • Find the area of a parametric surface. • Evaluate a surface integral as a double integral. • Evaluate a surface integral for a parametric surface. • Determine the orientation of a surface. • Understand the concept of a flux integral. • Understand and use the Divergence Theorem. • Use the Divergence Theorem to calculate flux.	**Teaching activities may include, but are not limited to:** • Direct Instruction • Cooperative Groups • Stations • Data Driven Instruction • Scaffolding

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• Understand and use Stoke's Theorem. • Use curl to analyze the motion of rotating liquid.	
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